Complex Roots

Problem 1

(a) Express $\frac{2}{1-i}$ as $a+ib$ and as $re^{i\theta}$ (where a, b, r, and θ are real).

(b) Find the real and imaginary parts, and the modulus and argument, of $e^{1+(\pi/3)i}$.

(c) Find all the fourth roots of -1. (The Mathlet `Complex Roots` may be useful in helping you to understand complex roots.)

(d) Find all complex numbers z such that $e^z = 1+i$.

Problem 2

This figlet gives a good insight into $(a+bi)^{\frac{1}{n}}$ and why there are n such numbers if $a+bi \neq 0$.

(a) For preliminary practice, click on Zoom, Angle, 4 (i.e., 4th roots); set the modulus slider to 1.00, and move the argument slider. The yellow dots are the 4th roots of the blue dot. Check out where the 4th roots of $1, i, -1, -i$ are. Play around with changing the modulus also. If you prefer, you can drag the blue dot around with the mouse.

(b) Using the figlet, given any integer $n > 2$, find experimentally by using the sliders the complex number α of smallest positive angle which is equal to one of its nth roots. Try this for different values of n, then make a cojecture expressing α in terms of n.

(c) Then prove your conjecture by complex number algebra.

Problem 3

(a) Choose 'root' = 6 and set the modulus slider to 1.0. Also click the 'zoom' button to zoom in. Now, when you move the angle slider, the cyan (light blue) z value vector rotates as do the green root vectors. When you rotate the argument through an arbitrary angle θ by how much do the root vectors rotate?

(b) Verify your answer in part **(a)** in the ofllowing case. Click the 'values' button so tha complex values of the 6 roots are shown. Keep the modulus slider at 1 and set the angle to 0. Write down the value of each root and compute its argument (angle). Now move the angle of z to $\pi/2$ radians and write down the value of each root and compute its argument. Finally, use a calculator to show these values satisfy your answer to part **(a)**.

(c) Using what you know about taking n^{th} roots, prove your answer in part **(a)**