

# Linear Phase Portraits: Cursor Entry

## Problem 1

(a) For a  $2 \times 2$  matrix  $A$  with rows  $[a, b]$  and  $[c, d]$  the trace is defined as  $\text{tr}A = a + d$ . Show that the characteristic polynomial of  $A$  is given by  $p(\lambda) = \lambda^2 - (\text{tr}A)\lambda + \det A$ .

(b) Use the visual to give a qualitative description (i.e. signs, real or complex, equal or not) of the eigenvalues in each of the colored regions in the trace-determinant plane. Also, include a copy of the  $TD$ -plane, labeling each region with the geometric type of the associated system  $\mathbf{x}' = A\mathbf{x}$ . Be sure to include the one dimensional regions given by the curves dividing the two dimensional regions.

(c) The red parabola divides the plane into 2 regions: above it  $A$  has complex eigenvalues and below it  $A$  has real eigenvalues. Use this fact to show that the equation of the red parabola is  $\det A = \frac{1}{4}(\text{tr}A)^2$ .

(d) Prove that the eigenvalues in the yellow region (in the second quadrant between the parabola and the positive  $y$ -axis) are complex with negative real parts. (Hint: you might want to use the description of the parabola given in part (c) and the formula for the characteristic polynomial in part (a)).

(e) Click somewhere in the yellow region in the second quadrant. Using the slider slowly increase the value of  $\text{tr}A$ . The matrix  $A$  should slowly move towards the yellow region in the first quadrant. Describe what happens to the phase portrait as  $\text{tr}A$  changes.