Linear Phase Portraits: Cursor Entry

Problem 1

(a) For a 2×2 matrix A with rows $[a, b]$ and $[c, d]$ the trace is defined as $trA = a + d$. Show that the characteristic polynomial of A is given by $p(\lambda) = \lambda^2 - (trA)\lambda + detA$.

(b) Use the visual to give a qualitative description (i.e. signs, real or complex, equal or not) of the eigenvalues in each of the colored regions in the trace-determinant plane. Also, include a copy of the TD-plane, labeling each region with the geometric tyoe of the associated system $\mathbf{x}' = A\mathbf{x}$. Be sure to include the one dimensional regions given by the curves dividing the two dimensional regions.

(c) The red parabola divides the plane into 2 regions: above it A has complex eigenvalues and below it A has real eigenvalues. Use this fact to show that the equation of the red parabola is $detA = \frac{1}{4}(trA)^2$.

(d) Prove that the eigenvalues in the yellow region (in the second quadrant between the parabola and the positive y-axis) are complex with negative real parts. (Hint: you might want to use the decription of the parabola given in part (**c**) and the formula for the characteristic polynomial in part (**a**)).

(e) Click somewhere in the yellow region in the second quadrant. Using the slider slowly increase the value of trA. The matrix A should slowly move towards the yellow region in the first quadrant. Describe what happens to the phase portrait as tr$A$ changes.