Mathlets in Lecture, Segment 1

Mathlets: An Introduction

Welcome to Module 1 of this short course. We'll be giving a number of examples of the use of the MIT Mathlets in various teaching situations. Several of these examples come from a presentation in July, 2013 at a workshop at MIT attended by high school teachers from around the world. Our first example is an introductory lecture addressed to a general audience of science teachers.

What I hope you get out of this video is that it's possible to give a pretty traditional lecture based on a Mathlet. What I hope the audience got out of this presentation was, well, the same thing. I was also trying to support the contention that mathematics is a highly visual and intuitive subject.

In using a Mathlet in lecture, it's important to realize that the students are much less aware that you are of the manipulations you perform. I tend to announce what I'm doing when I grab a slider or push a button. The students can't see that maneuver. All they can see is the effect.

Also, many of the Mathlets carry a lot of information. Students don't comprehend it all at once. In fact, at first they understand nothing of what's on the screen. You have to lead your audience through the various visual components with care. Watch how I do this in the following video and see if I pace it right for you.

So I'm going to model-- I'm going to begin by modeling how I might use some of these applets in a lecture. And I'm going to go to clear down to the bottom here and go to Phase Portraits, Cursor Entry.

This is a nice picture. For the cognoscenti, this models a homogeneous linear system of differential equations. This whole thing is controlled by a matrix. And you can see the matrix down at the bottom here. I called it A. And it these particular values right now.

A 2x2 matrix determines a couple of numbers. It can take the sum of the diagonal entries. That's called the trace. It also has a number called the determinant. And those two numbers determine most of the pattern that you see here. They control the dominant features of this pattern.

By changing the trace and determinant, I will change the way the pattern looks. And you'll see that. I can adjust the matrix by moving its trace. You can see what happens. The pattern of trajectories changes when I do that. Or I can change the determinant using the slider over here. And that also changes the pattern of trajectories.

In fact, I can change both the trace and the determinant together with a single control in the middle here. So this little plus sign in the trace determinant box is alive. And I can move through this trace determinant space by just pulling that cursor around. And you get quite an active display of the different trajectories that occur.

So far I've just been inside of this little part of the parabola. Now, I'm going to start to move, explore more widely and push over across this vertical line. So here, the spirals get tighter and tighter and tighter and tighter until when you hit that vertical line, they become circles. These are periodic orbits, in the parlance.

If I continue to move over, the trace is negative. I have a stable system, as they say. Now the trajectories are running in. You can see the little arrows on the trajectories. They're running in towards the origin. They're moving around here.

Now, I'm going to come down here towards the green section. And notice that the trajectories are all yellow. I'm in the yellow section of the parabola right now.

If I push down through here, there's this marginal case that happens. It's red. And then that's followed by another completely different pattern of trajectories. This is called a node. And again, it has different shapes.

And if I continue to push down here, I get another marginal case, a comb figure. And then down here, these are called saddles. And there's a great variety of different saddles that I can see by wandering around randomly in that blue area underneath. And so then I can complete my tour by going back through the unstable nodes and back to unstable spirals, which is where we began.

So the student can see-- Now, a student can solve these things. The student can solve these equations. They solve these equations by finding eigenvectors. And there's a complicated procedure that you do with a piece of paper and pencil or Matlab. And when you do it, you find that different kinds of functions get involved in the solutions.

For example, these spirals involve sines and cosines. These nodes down here do not. They just involve exponentials. And the saddles just involve exponentials.

And so students are prone to think these are completely different situations with no relationship to each other. But one of the things this tool does is to let you see the smooth morphing as you move from one section to another. There's no jumps. It's a smooth motion. They're all continuously connected to each other. And that's part of the lesson. So there are a lot of lessons here that are extremely difficult to convey with chalk or talk in a lecture situation. And you can imagine sending students home to conduct various exercises with something like this.

I chose to begin with this applet because it's visually dramatic. I thought it would be a good introduction to the applets for a general audience, despite being mathematically quite advanced. It's also a great example of a Mathlet which illuminates some piece of mathematics far better than any static mechanism could do.

Here are some things I'd like you to think about arising from this little lecture fragment. First, when you first saw the Mathlet, there was a lot of information on the screen. Do you remember how you felt? Were you confused by it? I ask this because your student's experience will be not too different from your own, unless you've seen an applet like this before.

Second, I introduced the various windows and functionalities of the Mathlet over a period of several minutes. Make an ordered list of these in sequence and think about other ways I might have presented them to this audience. Third, there are still other aspects of this tool, which I didn't talk about at all. Suggest some additional investigations one can pursue using it.