

Coupled Oscillators

Work with the two mass system shown on the *Coupled Oscillators* Mathlet <http://math.mit.edu/~jmc/18.03/coupledOscillators.html> The Theory page accompanying the Mathlet analyses this in general and works out the special case in which the masses and the spring constants both coincide. In this exercise you'll work this out for equal spring constants (take $k_1 = k_2 = k_3 = 1$), but now with different masses: $m_1 = 2$ and $m_2 = 1.25$.

Begin by experimenting with the Mathlet. It's pretty easy to figure out. You can set the initial velocities by switching on v_1 and v_2 and grabbing the end of the velocity indicator. Set initial velocities to zero, and check them often to see that they are still zero. I recommend switching the velocity curves off when you've finished setting their initial values. But watch out; manipulating other features can reset the initial velocities. You can tell by checking the slopes of the solutions at $t = 0$.

Further note: The sliders only allow a discrete set of values, but the allowed values depend upon the zoom setting. In any case, getting the values within 0.01 of the requested values will be good enough.

(a) Find two normal modes on the Mathlet. Is one in synch and one 180° out of synch, as they were in the equal mass case? In each case, use the crosshairs and readout of coordinates on the Mathlet to measure the amplitudes of the two sinusoids. In each case, write A_1 for the amplitude of the first mass and A_2 for the amplitude of the second, and compute the ratio A_2/A_1 . Then measure the period of these sinusoids and record it.

(b) Now write down the equations of motion ($m_1\ddot{x}_1 = \dots$, $m_2\ddot{x}_2 = \dots$) and the 4×4 "companion matrix." Write it in block form as was done in the Theory page.

(c) Use the trick discussed in the Theory page to find the eigenvalues. What periods do you expect your normal modes to have? Compare with your measurements.

(d) Find the corresponding eigenvectors, write down the two normal modes. (I mean: write down the sinusoidal solutions. Write them in the form $\mathbf{x} = A \cos(\omega t - \phi)\mathbf{v}$, where \mathbf{v} is a constant vector and ω is a positive number, and A and ϕ can be anything.)

Determine the ratio A_2/A_1 from this computation, and compare with your measurements.