

Graphing Rational Functions

1. The MIT Mathlet **Graphing Rational Functions** relates the graph of a rational function with its algebraic expression, which is of the form

$$f(x) = \frac{\text{a polynomial}}{\text{another polynomial}}$$

To understand the behavior of $f(x)$ it is useful to factor each of the polynomials. This Mathlet assumes that you can factor each into linear factors with real coefficients, and that all factors are of the form $(x - a)$: so an example would be

$$f(x) = \frac{(x - 2)(x - 3)}{x + 1}$$

The values $x = 2$ and $x = 3$ in this example are the *zeros* of $f(x)$ —they are the values of x for which $f(x) = 0$. The value $x = -1$ is a *pole* of $f(x)$ (and is the only pole in this example). Here the denominator becomes zero (and the numerator is not zero), so the function blows up to plus or minus infinity near $x = -1$.

2. Open the Mathlet. The zeros of the displayed rational function are marked by small o's on the number line at the bottom of the screen, and the poles are marked with small x's. (Think of this as the top of an arrow pointing up at you.) Can you guess the formula for this rational function? Check your guess by invoking the [Show Function] button.

Now grab the zero by positioning the cursor near it in the number line and depressing the mousekey. You can move it around by dragging it. (Position poles the same way.) Drag the zero slowly from $x = -1$ to $x = +3$ and watch what happens. There's one special value of x in between. Describe what you see for that special value, and why the graph looks the way it does.

3. Look at the formula for the rational function $f(x) = (x + 1)/(x + 3)$ and compute $f(97)$. When x is very large, what is the approximate value of $f(x)$? Is your calculation borne out by the applet? Notice that you can zoom in or out. Zoom back so that the range from -6 to 6 is shown on the graping window. Make sure the formula is $f(x) = (x + 1)/(x + 3)$ and unselect [Show graph] and [Show Formula]. Add another zero to to the rational function using the [Add zero] radio button and then selecting a location for it on the number line. By looking at the number line, which should show two zeros

and a pole, write down your best guess about the formula for the function. Check your work by selecting formula.

4. Now take a piece of paper and do your best to sketch a graph of this function. When you are happy with it, select [Show graph] and compare results.

Here are some things to notice. For these comments I will put the second 0 at $x = 1$. You will probably have a used a different zero, so set $f(x) = (x + 1)(x - 1)/(x + 3)$. The visualization will be helped if you select [Show Vertical Asymptotes].

(a) When x is large, the value of this function is close to x . The graph doesn't level out and become flat as the earlier case did. Instead the graph is asymptotic to the line $y = x$. Please explain why! Suppose that your function had four zeros and two poles. What would the behavior be for very large positive x ? How about for very large negative x ?

(b) As x crosses a pole, the graph becomes asymptotic to a vertical straight line. Please explain why!

(c) As x decreases, $f(x)$ changes sign whenever x crosses either a zero or a pole. Please explain why!

5. These three observations let you draw a pretty accurate graph of any rational function. For example, take a piece of paper and use the three steps to sketch a graph of the rational function

$$f(x) = \frac{(x + 2)x}{(x + 1)(x - 1)}$$

When you are happy with it, create this function on the Mathlet and check your answer. Where you and the Mathlet differ, explain why.

6. The statement (c) needs to be refined slightly. To see this, think about the rational function $f(x) = 1/(x - 1)^2$. It's always positive! By thinking about what happens for large x and as x gets near zero, sketch a graph. The function is *even*, $f(-x) = f(x)$, so its graph is symmetric about the y axis. To check your work, set up the Mathlet with two poles. Put one of them at $x = 1$, and then move the other towards it. Describe what happens! Compare with your answer.