

Riemann Sums

This Mathlet explores various versions of the Riemann sum approximation of a definite integral.

1. Under what conditions on the function $f(x)$ is the Min sum equal to one of the Evaluation point sums? How about the Max sum?

2. What is the gap between the left endpoint evaluation and the right endpoint evaluation? Take $n = 8$, for example, and study how the two sets of rectangles are related to each other. Can you explain why the gap is what it is observed to be? Notice that rolling the cursor over the graphing window produces a readout of the coordinates. Make a prediction about the size of the gap when $n = 20$, and check your prediction.

3. Invoke Simpson's rule. Take $n = 1$ and look at the effect for each of the functions $f(x)$. The red graph represents the approximation to $f(x)$ used in Simpson's rule. What do you observe about the graph of the function and the red graph? Please explain. Check your answer by setting $n = 2$ and repeating the observations.

Simpson's rule is particularly good when $f(x) = x^2 - 2x$. Please explain.

4. Choose the menu item $f(x) = x^2 - 2x$. Make a table with rows labeled 3, 6, 12, 25, 50, 100, 200, and columns labeled "Ev mid, Min, Max, Trap" and fill it in.

What do you reckon the value of the integral is?

For each of these ways of forming a Riemann sum, the estimate gets better by approximately a factor every time you double the number of intervals. What's that factor, roughly?